REVIEWS

Laminar Boundary Layers. Edited by L. ROSENHEAD. Oxford University Press, 1963. 688 pp. £4. 10*s*.

The subtitle reads: 'An account of the development, structure and stability of laminar boundary layers in incompressible fluids, together with a description of the associated experimental techniques'. That is a fair description, except to add that there is also a chapter on flow at small Reynolds number. The authors are L. F. Crabtree, G. E. Gadd, N. Gregory, C. R. Illingworth, C. W. Jones, D. Küchemann, M. J. Lighthill, R. C. Pankhurst, L. Sowerby, J. T. Stuart, E. J. Watson, and G. B. Whitham.

This large book—supplemented by the companion volume *Incompressible Aerodynamics* edited by Thwaites, and a possible forthcoming one on turbulence—represents the long-awaited successor to Goldstein's *Modern Developments in Fluid Dynamics*. The King James Bible is said to be the only work of

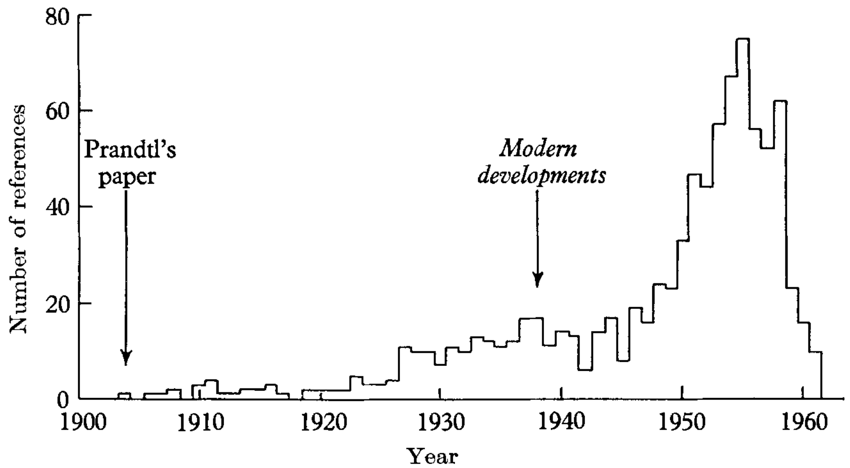

FIGURE 1. Number of references cited in *Laminar Boundary Layers* in each year since Prandtl's 1904 paper.

art ever produced by a committee, and *Modern Developments* holds the same position in our little world of fluid mechanics. Now another committee has presumed to rewrite that classic, and we naturally ask: was it necessary? did they succeed? will the new work altogether supplant the old?

The need for revision is suggested by simple statistics on cited references. A splendid feature of the book is that these have been gathered into a single list arranged alphabetically by author. This eliminates irritating *loc. cit.*'s and provides a bibliography that no devotee of boundary layers can scan without resolving to look up an unfamiliar or forgotten paper. Of the 900-odd references, more than three-quarters were published after *Modern Developments* appeared (figure 1). Of course the publication explosion of the 1950's seen in the figure is somewhat misleading. Goldstein, too, cited almost 900 works, of which only about one-fifth survive in the new list; and time will likewise blessedly erode the peak of 75 essential papers on boundary layers published in 1955.

The apparent decline in production three years later simply means that the days of Durand and Goldstein are past; large collaborative works in fluid mechanics can no longer be completed in reasonable time in English-speaking countries. (We still wait, after eight years, for Lagerstrom's valuable article on viscous flow to appear in the Princeton series.) Although the book is copyrighted 1963, and the preface dated 1961, the cut-off date for authors was evidently about 1958. Most of the few later references are due to Stuart, who apparently waged a heroic struggle to maintain timeliness—including a stop-press list of additional references squeezed on to the last unnumbered page in proof.

The first two introductory chapters are the most unconventional, despite their superficial parallelism with *Modern Developments*. Lighthill writes in his celebrated 'masterly' style, beginning with d'Alembert's paradox and—after a survey of physical properties of air and relevant dimensionless groups—concluding with the resolution of that mystery in the fact that Reynolds number is 'the liveliest of all non-dimensional parameters'.

His second chapter is a *tour de force* devoted to the proposition that vorticity is the key to understanding boundary layers. That viewpoint is pursued relentlessly, vorticity being invoked to explain displacement thickness, separation, instability, transition, and turbulence. The reader is charmed by persuasive arguments: the act of blowing out a candle at a distance can scarcely be understood except through vorticity. However, the unsophisticated student might find an easier introduction in the less elegant but remarkably clear first two chapters of *Modern Developments*. That much, at least, of Goldstein must be preserved; we need his essay as well as Lighthill's. A victim of the 1958 cut-off on references is the standard graph, on page 106, of drag versus Reynolds number for a circular cylinder, which would have benefited from the addition of the careful measurements of Tritton (1959; cited for other purposes by Stuart) in conjunction with the improved theory of Kaplun.

Whitham derives the Navier–Stokes equations, and provides an organized survey of the more useful exact solutions. These include a little-known favourite of mine: Kovásznay's 'wake behind a grid'. For the Jeffery–Hamel flow in a wedge, an ingenious dynamical interpretation helps to cut through the mathematical details.

Flow at low Reynolds number has seen recent dramatic discoveries that led to final resolution of the paradoxes of Stokes and Whitehead, and intriguing work on the swimming of micro-organisms. Illingworth surveys the subject in straightforward fashion. I would demur only at his reluctance to criticize. For example, it ought to be recorded that Goldstein's term in R^5 in the Oseen drag of a sphere (p. 177) is erroneous, and was corrected by Shanks (*J. Math. Phys.* 1955); and that Carrier and Lin's solution for the leading edge of a flat plate (p. 181) is incorrect, and moreover cannot be matched with the boundary-layer solution. It would also be useful to note on p. 181 that the most general solution of the Oseen equations in closed form is for the elliptic paraboloid (Wilkinson, *Quart. J. Mech. Appl. Math.* 1955).

Jones and Watson's discussion of two-dimensional boundary layers covers such recent work as conditions at separation, the role of co-ordinate systems, and higher approximations for the flat plate. The last could be improved in some respects, beginning with a reminder to a sea-faring race that Kuo's finite plate (p. 229) lies *along* rather than *athwart* the stream. It is useful to recognize the second term in Imai's expansion (p. 229) as representing a concentrated force at the leading edge; and to realize that its neglect invalidates Kuo's result. Again (p. 231) the Carrier–Lin solution cannot be matched to that of Blasius.

I applaud the authors' decision to write Blasius's problem ($f''' + ff'' = 0$, and so on) in its canonical form, free of factors of 2 in either the equation (Prandtl, Schlichting) or the outer boundary condition (Goldstein). But then the remark that $f'(\infty) = 2$ in other work, repeated on page 242, is not only unnecessary but confusing. On the other hand, I deplore another and more far-reaching change in notation. Following Thwaites, the authors and editor have—alas!—chosen to denote the co-ordinate normal to the surface by z rather than the time-honoured y, even in two dimensions. This, they say, 'was agreed in order to facilitate transition to three-dimensional theory in the form conventionally adopted for aircraft wings'. But obviously the latter notation is at fault, and was long ago corrected by our Soviet colleagues, whose z-axis runs out along the span of the wing. Among other evils, the scheme adopted here requires using an impossible squiggle to denote the complex variable $x + iz$ in plane flow (pp. 184, 234). We must all oppose this innovation; and I am delighted to see that Whitham has, on page 115 and again on page 143, already slyly begun the rebellion under the very nose of the editor.

This book was written at the dawn of the computer age. Consequently, although Gadd, Jones and Watson provide an admirable review of approximate methods of solution for two-dimensional boundary layers, one must suppose that the subject would be treated quite differently in a few years. Already Flügge-Lotz (*Z. angew. Math. Phys.* 1958) and Smith & Clutter (*A.I.A. Aerospace J.* 1963) have shown the effectiveness of electronic computation for plane and axisymmetric boundary layers, and their techniques will undoubtedly supplant much earlier work.

Stuart contributes chapters on unsteady boundary layers and on hydrodynamic stability, and both are superb. The latter, a miniature monograph of 90 pages, would seem the ideal introduction for a student coming fresh to the field. Both subjects are absent from *Modern Developments*, as are three-dimensional boundary layers. Crabtree, Küchemann and Sowerby give a useful derivation, using vector operators, of the three-dimensional equations. Rotationally symmetric flows are examined in detail, followed by what is known of general three-dimensional cases.

The final chapter, on experimental methods, by Pankhurst and Gregory, seems to dangle from this book as an afterthought, whereas its counterpart formed an integral part of *Modern Developments*, because it served to introduce a number of chapters full of experimental results. It seems that space was also severely limited. Thus, for example, the schlieren method is only mentioned in passing, whereas Goldstein found room for two pages of description, a diagram, and sample photographs.

We may conclude that this new book is a success even by the hard test of comparison with its classic predecessor. It preserves and codifies the essentials of older work, while discussing the new in remarkable variety. (I would only wish, off-hand, for *wall jet* to be added to the index.) In particular, one finds at least brief description of significant new developments, such as optimal co-ordinates and the treatment of flows at low Reynolds number by the method of matched inner and outer expansions. Incidentally, the latter could profitably have been extended also to high Reynolds number. The standard of writing is uniformly good. The book is plentifully and attractively illustrated. And the references are reasonably cosmopolitan; though one is amazed, upon leafing through Prof. Loitsianskii's new Russian book on laminar boundary layers (1962), to see how much more Slavic the same subject looks from Leningrad. I have decided not to discard my copy of *Laminar Boundary Layers* when it is superseded by the 1988 version, but to keep it on the shelf next to *Modern Developments*.

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